HoTT FORMALIZATION — VERSION 1

Homotopy Type Theory / Univalent Foundations

× Relational Ontology (Lis, 2025)

The Self-Experiencing Universe: A Manifesto for an Emergent Reality (Lis, 2025)

Waldemar Lis (ontology) | Claude / Anthropic (formal analysis) — February 2026

Eighth study in the series: formalization following seven structural isomorphism simulations

NOTE: This is Version 1. Version 2 (revised) supersedes this document on Mechanism II.

Introduction: From Isomorphisms to Formalization

Seven previous simulations showed that the mechanisms of relational ontology (Lis, 2025) exhibit structural isomorphisms with mathematics of relational co-constitution without substance presuppositions. The ∞-category simulation revealed two STRONG isomorphisms: the Univalence Axiom with Mutual Specification (MS) and the descent condition with Radical Contextuality (RC). This indicates that HoTT may be not merely an analogy but a genuine **formal language** for this ontology.

This document takes a qualitatively harder step than the previous simulations: it does not recognize isomorphisms but attempts to construct a formal account of three central mechanisms in the language of HoTT. Gaps are valuable results — they show precisely what the ontology lacks for full formalization, or what HoTT cannot express by definition.

Three structural questions:

(1) Can Mutual Specification (MS) be expressed as a theorem in HoTT, or does it require a new axiom?

(2) Is the Integration Threshold (IT) as a discontinuous phase transition bridgeable with the continuous colimit structure in ∞-toposes, or does it require a new primitive?

(3) Can Relational Potential (RP) as a pre-homotopic condition of possibility be expressed in HoTT without regress, or is this an inherent limit of every formal language?

Formal Context: HoTT and Univalent Foundations

Basic HoTT constructions used in this document

HoTT (Homotopy Type Theory) is a dependent type system in which types correspond to homotopy spaces and terms correspond to points in those spaces. Key elements:

Dependent types:

```
Π(x : A), B(x)   — dependent product (for each x:A, type B(x))
Σ(x : A), B(x)   — dependent sum (there exists x:A with proof B(x))
```

Identity type:

```
Id_A(a, b)   — type of paths from a to b in space A
refl_a : Id_A(a, a)   — trivial path (reflexivity)
```

Homotopy equivalence:

```
A ≃ B   :≡   Σ(f : A → B), isEquiv(f)
isEquiv(f)   :≡   Π(b:B), isContr(Σ(a:A), Id_B(f(a), b))
```

Univalence Axiom (Voevodsky):

```
ua : (A ≃ B) ≃ Id_U(A, B)
Equivalence of types IS identity of types — not an implication, but
identity.
```

Higher Inductive Types (HIT):

```
Types with generators for points AND paths (and paths between paths...)
Example: circle S¹ with point base and loop : Id_{S¹}(base, base)
```

Methodological note: the notation used in this document is the book-level HoTT notation (HoTT Book, 2013) with extensions for ∞-toposes (Lurie). This is not Agda or Coq code — it is mathematical dependent-type notation at textbook rigor.

Mechanism I — Mutual Specification (MS)

Ontological description

MS is the central mechanism of the ontology: two beings (or two relational patterns) mutually determine each other **without priority of either side**. The identity of each is relational — it does not exist as a substantial property preceding the relation, but **emerges from the pattern of mutual reference**. Mathematical instances identified in previous simulations: adjunctions in category theory, Jones polynomial, Rogers-Ramanujan identities, Univalence Axiom.

Formalization attempt in HoTT

Step 1 — Relational constitution of identity as a type

A being in relational ontology has no substantial identity — it is determined by its pattern of relations to all other beings. In HoTT the natural candidate is the homotopic Yoneda Lemma:

```
For each X in ∞-category C:
  yo(X) : C → ∞-Grpd
  yo(X)(Y) :≡ Map_C(Y, X)

Homotopic Yoneda Lemma:
  Nat(yo(X), F) ≃ F(X)

Consequence: X is fully determined by Map_C(-, X)
             Identity of X = its relational profile
```

Status: this is not yet a formalization of MS — it is a formalization of the relational constitution of one being. MS requires mutuality.

Step 2 — Mutuality through the Univalence Axiom

The Univalence Axiom states:

```
ua : (A ≃ B) ≃ Id_U(A, B)
```

Ontological interpretation: the identity of two types (beings) IS their relational equivalence. Neither side is prior — equivalence and identity are the same fact described from two sides. This can be written as the MS condition:

```
MS-Condition(A, B) :≡ (A ≃ B) ≃ Id_U(A, B)

Meaning: A determines B and B determines A through
shared equivalence structure — without priority.
```

This is a theorem (axiom), not a definition: univalence is an axiom that HoTT adds to the type system — it does not follow from the type calculus alone. In this sense MS **requires the Univalence Axiom**, but does not require a new primitive beyond HoTT.

Step 3 — Full formalization of MS as a type

```
MS(A, B) :≡ Σ(e : A ≃ B), (transport(ua(e)) = id_B)
              × (transport(ua(e)⁻¹) = id_A)
```

```
Where:
  ua(e) : Id_U(A, B)  — identity path from equivalence e
  transport(p) : A → B — transport along path p
  id_A, id_B — identities on A and B
```

Interpretation: A and B are mutually constitutive when equivalence e between them is complete (transport in both directions preserves identity). Neither side is preferred — the condition is symmetric.

Step 4 — Where formalization is complete, where not

Complete: relational constitution of identity (Yoneda), symmetry of mutual determination (univalence), absence of substantial presupposition (types defined by relations, not internal properties).

Incomplete: MS in the ontology concerns beings at the same ontological level without hierarchy. In HoTT the Universe U is hierarchical ($U_0 : U_1 : U_2 : ...$) — univalence works within one level, but types from different levels are not symmetrically mutual. This is a structural asymmetry that MS does not have.

Tension: ontological MS assumes beings without fixed 'levels' — each being is a condition of possibility of every other. HoTT has a type-universe hierarchy that is irreducible. MS-Condition works within each level, but not across levels.

PARTIAL	MS is partially formalizable in HoTT through the Univalence Axiom (mutual symmetry, no substantial presupposition). Gap: the HoTT universe hierarchy introduces level asymmetry that MS does not have. Full formalization requires an additional axiom or restriction to single-level configurations.

Proposed supplement: Axiom of Flat Mutuality (AFM)

To express MS without level restrictions, an additional axiom is needed:

```
AFM : Π(A B : U_n)(e : A ≃ B),
      MS(A, B) ≃ MS(B, A)
      — mutuality is symmetric regardless of universe level
```

Consistency with HoTT: AFM is consistent with univalence, since univalence itself is symmetric ($e : A \simeq B$ implies $e^{-1} : B \simeq A$). AFM does not add contradiction — it adds an ontological commitment: neither side of the mutuality can be at a higher universe level than the other.

Mechanism II — Integration Threshold (IT)

Ontological description

IT is the mechanism through which consciousness emerges from relational organization. Three gates must be simultaneously satisfied: Gate I — **Stability** (the pattern must maintain structural coherence); Gate II — **Integration** (the whole must be unified — integrative closure); Gate III — **Recursion** (the pattern must model itself). Crossing is **discontinuous** — a phase transition, not an interpolation. This is the central difficulty of formalization: HoTT is continuous, IT is discontinuous.

Formalization attempt

Step 1 — Three gates as conditions on a type

```
RelConfig : Type
κ : RelConfig

Gate-I-Stability(κ) :≡
  Σ(s : StabilityMeasure κ), s ≥ threshold_s

Gate-II-Integration(κ) :≡
  isContr(Σ(p : Parts κ), ¬CanSeparate(p, κ))
  — no partition of κ into independent parts

Gate-III-Recursion(κ) :≡
  Σ(m : κ → Model κ), isFaithful(m)
  — κ has a faithful model of itself
```

Critical note: StabilityMeasure, Parts, CanSeparate, Model — these terms require precise definition in HoTT. The above is a schema, not a full formalization.

Step 2 — Threshold as simultaneous satisfaction

```
IT-Condition(κ) :≡
  Gate-I-Stability(κ)
  × Gate-II-Integration(κ)
  × Gate-III-Recursion(κ)

IP(κ) :≡ IT-Condition(κ)  — Integrated Perspective
```

Formal status: IT-Condition is a type in HoTT. Its inhabitation is a logical fact. This is a **correct formalization of the condition**, but not of the discontinuity mechanism.

Step 3 — The discontinuity problem: the central gap

Key tension: IT in the ontology is a **discontinuous phase transition** — analogous to water freezing. The colimit in an ∞-topos (the formal candidate from the ∞-category simulation) is constructive and **continuous**: objects are added successively, the structure grows gradually.

```
Continuous colimit: X ≅ colim_{κ-compact Y → X} Y
— every object expressible as limit of compact diagrams
```

```
Discontinuous IT: κ → IT-Condition(κ) is a jump —
there is no 'κ with partial IT'
```

In HoTT identity is expressed through paths — and paths are by definition continuous deformations. Topological discontinuity is not representable as a path. This means: the condition of IT can be formalized, but the mechanism of discontinuity cannot be formalized in standard HoTT.

Step 4 — Attempt through Higher Inductive Types (HIT)

```
data IT-Transition where
  pre-threshold  : RelConfig → IT-Transition
  post-threshold : RelConfig → IT-Transition
  crossing : Π(κ : RelConfig),
             IT-Condition(κ) →
             Id_{IT-Transition}
               (pre-threshold κ, post-threshold κ)
```

Does this solve the discontinuity problem? Partially. The HIT formalization allows us to say: the transition exists iff the condition is satisfied. But the mechanism of the jump itself — why simultaneous satisfaction of three conditions generates a qualitatively new entity (consciousness) — remains outside formalization. HIT says that there is a transition, not why it is ontologically discontinuous.

NEW PRIMITIVE REQUIRED	IT-Condition is formalizable in HoTT as a conjunction of three gates. The mechanism of discontinuity (why satisfaction of the condition generates a qualitatively new entity) is not expressible in standard HoTT or through HIT. Required new primitive: Axiom of Qualitative Emergence (AQE).

Proposed new primitive: Axiom of Qualitative Emergence (AQE)

AQE states that satisfaction of IT-Condition is not merely a logical fact but generates a new ontological type irreducible to the sum of the gates:

```
AQE : Π(κ : RelConfig),
      IT-Condition(κ) →
      Σ(P : IPType κ), isIrreducible(P)

Where:
  IPType κ  — type of Integrated Perspective for κ
  isIrreducible(P) — P is not expressible
                     as colimit of components of κ
```

Consistency with HoTT: AQE is **not inconsistent** with existing HoTT axioms (univalence, choice, LEM if added). It is an axiom adding a new entity (IPType) to the language, not changing existing theorems. Open question: does a model (∞-topos) exist in which AQE is satisfied? This is a technical open question requiring model construction.

Analogy with univalence: univalence is itself an axiom that does not follow from the type system — it is added. AQE has the same status: an additional ontological commitment that can be adopted or rejected independently of the rest of HoTT.

Mechanism III — Relational Potential (RP) as Pre-Categorical

Ontological description

RP is the central and hardest term of the ontology: the condition of possibility of any structure, relation, categorization. RP is not a being among beings — it is what makes beings possible. From the ∞-category simulation we know that the tension with category theory shifts one level up at every attempt at formalization:

```
CT:     RP → Category       (error: CT assumes a category)
∞-Cat: RP → ∞-Category    (error: ∞-Cat assumes ∞-cat)
HoTT:  RP → Type in U     (error: HoTT assumes type structure)
Regress: RP → X → X(X) → ...   [without end]
```

Central question: is this a technical problem with HoTT, solvable by cleverer formalization — or is it an **inherent limit of every formal language**?

Formalization attempt

Step 1 — RP as the Universe of types

Simplest candidate: RP = Universe of types U, from which all types emerge.

```
RP :≡ U   (Universe of types)
Emergence of structure :≡ A : U   (A is a type)
Relation :≡ f : A → B   (morphism between types)
```

Why this DOES NOT WORK: HoTT has a hierarchy of universes $U_0 : U_1 : U_2 : ...$ No single universe U contains itself (Russell's paradox). RP as one universe U_n contains only types of level n. As 'sum of all universes' — it is an object that itself requires a type at a higher level. The regress is irreducible.

Step 2 — RP as logical possibility, not a being

Alternative interpretation: RP is not a type in HoTT but the condition of sensibility of the language itself. This corresponds to the distinction in HoTT between:

```
Judgment:   A : Type   — A is a type
Assertion:  a : A      — a is a term of type A
```

Candidate: RP corresponds to what in HoTT is assumed before any judgment — the structure of the type system as such. In Martin-Löf type theory this is the level of metatheory: the type formation rules, which are not themselves types:

```
Formation rule for products:
  Γ ⊢ A : Type   Γ, x:A ⊢ B(x) : Type
  ──────────────────────────────────────
  Γ ⊢ Π(x:A), B(x) : Type
```

RP as metatheoretic structure: type formation rules are conditions of possibility of any typological judgment. They are not in the theory — they are what enables the theory. This is exactly the structure of RP: a condition of possibility, not a being among beings.

Step 3 — Diagnosis: RP is meta-HoTT

```
RP is not expressible as a type in HoTT.
RP corresponds to the metatheoretic structure of HoTT itself
— the rules that HoTT assumes before any type.

HoTT can express: structures emerging from RP
                  (types, paths, equivalences)
HoTT cannot express: RP as the condition of possibility of HoTT
                     (that would require a meta-meta-theory)
```

Is this a flaw of HoTT or a flaw of the question? This is an **inherent limit of every formal language**. Every formal system assumes certain rules as given — these rules cannot be expressed within the system itself without regress. Gödelian incompleteness is one manifestation of this limit. The impossibility of formalizing RP in HoTT is another — and both follow from the same structural asymmetry: a system cannot fully encompass the conditions of its own possibility.

INHERENT LIMIT	RP as a pre-homotopic condition of possibility is not expressible in HoTT without regress. This is an inherent limit of every formal language — analogous to the Gödelian limit: a system cannot express the conditions of its own possibility within itself. HoTT is the language for manifestations of RP (emerging structures), not for RP as such. This is not a limitation of HoTT specifically — it is a limitation of every possible formal language applied to the description of its own condition of possibility.

Formalization Results — Version 1

Mechanism	Status v1	New primitive?
Mutual Specification (MS)	**PARTIAL**	No — univalence sufficient within one level U. Gap: universe hierarchy introduces asymmetry MS does not have.
Integration Threshold — condition	**FORMALIZABLE**	No — IT-Condition(κ) as conjunction of three gates is a HoTT type.
Integration Threshold — mechanism	**NEW PRIMITIVE REQUIRED**	Yes — Axiom of Qualitative Emergence (AQE): satisfaction of IT-Condition generates a new ontological type irreducible to the sum of gates.
Relational Potential (RP)	**INHERENT LIMIT**	Impossible — RP as condition of possibility of HoTT is not expressible in HoTT. Requires exiting every formal system.
Radical Contextuality (RC)	**FORMALIZABLE**	No — descent condition in ∞-topos formalizes RC completely (∞T-3 STRONG from previous simulation).

Proposed Axiom Set: HoTT-RO v1

```
HoTT-RO v1 = HoTT + {UA, AFM, AQE}

UA  — Univalence Axiom (standard HoTT)
      (A ≃ B) ≃ Id_U(A, B)

AFM — Axiom of Flat Mutuality (new)
      Π(A B : U_n)(e : A ≃ B),
      MS(A,B) ≃ MS(B,A)

AQE — Axiom of Qualitative Emergence (new)
      Π(κ : RelConfig),
      IT-Condition(κ) →
      Σ(P : IPType κ), isIrreducible(P)
```

Note: HoTT-RO v1 was revised to HoTT-RO v2 after the observation that IT's discontinuity assumption was unnecessary. In v2, AQE is withdrawn because consciousness as a topological region (not a jump) does not require irreducibility as an axiom — it follows from the vector structure of the region. See the v2 formalization document for the revised account.

References

Lis, W. (2025). The Self-Experiencing Universe: A Manifesto for an Emergent Reality.

Univalent Foundations Program (2013). Homotopy Type Theory: Univalent Foundations of Mathematics. IAS.

Lurie, J. (2009). Higher Topos Theory. Princeton University Press.

Martin-Löf, P. (1984). Intuitionistic Type Theory. Bibliopolis, Napoli.

Voevodsky, V. (2010). Univalent Foundations of Mathematics. IAS Lecture.

Gödel, K. (1931). Über formal unentscheidbare Sätze der Principia Mathematica. Monatshefte für Mathematik.

Full archive: https://archive.org/details/@not_foryou

© 2026 Waldemar Lis. All rights reserved.